Dynamics and Properties of Chiral Cosmic Strings

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Abstract

Chiral cosmic strings naturally arise in many particle physics models, in particular in supersymmetric theories with a D-term. These strings have a single fermion zero mode in the core. We derive the general equation of motion for such strings. In Minkowski space we give the self-intersections for an arbitary varying current on the loop, showing that the self-intersection probability is dominated by the fraction of loop with maximal charge. We show how to relate the charge to the fermion condensation temperature, arguing that strings which become current carrying at formation will automatically have a maximal charge. Any daughter loops produced are likely to have the same charge as the parent loop. Possible models for chiral cosmic strings are also discussed and consequences for D-term inflation mentioned.

1 Introduction

Recent attempts to use topological defects, and in particular cosmic strings [1], for structure formation have focussed on mixed scenarios including both strings and inflation [2]. The Nambu-Goto action was used to model the effect of the cosmic strings. This effective action results from the abelian Higgs model. However, in general cosmic strings formed in supersymmetric theories have fermion zero modes in the core [3], rendering the string current-carrying. Supersymmetric models where a U(1) gauge symmetry is broken via a Fayet-Iliopolous D-term can lead to a period of hybrid inflation [4, 5]. Cosmic strings would then form at the end of inflation, and strings formed in this way will have a single fermion zero mode in the core, either left or right moving, giving rise to a chiral current [3]. Such a string is commonly referred as a chiral string. The effective action of such a string is not the Nambu-Goto action, but the chiral action [6].

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Such strings are particularly interesting because they will have increased longevity, since the angular momentum of the charge tends to counterbalance the tension of the loop. A loop which is completely stabilised classically is a vorton state [7]. The existence of vortons puts severe constraints on the underlying particle physics theory [8]. Unlike superconducting strings [9], in the case of chiral cosmic strings the current is not electromagnetic. Indeed, the fermion zero mode cannot be electromagnetically coupled due to anomalies [6].

In a previous work [10] it was shown that chiral strings carrying maximal or near-maximal current never self-intersect, and thus rapidly form vorton states which remain the same size as at formation. These will in general be much larger than the $\mathcal{O}(100)$ string widths of those produced from ordinary superconducting cosmic strings [7]. These vortons will scale as ordinary matter, and thus may potentially come to dominate the matter content of the Universe. Indeed, it has previously been shown that this leads to very stringent constraints on the underlying particle physics theory [11].

For general current-carrying strings, loops are characterised by two independent quantum numbers N and Z, both of which are conserved. For strings with fermionic zero modes, these are associated with combinations of the left and right moving currents. In the chiral case there is obviously only one independent quantum number, formed from the null current.

The equations of motion were derived in Minkowski space in [10]. In section 2 we re-derive the equations of motion for a chiral cosmic string, generalising them from the form given in [10], to that of a general background space. The equations are then considered in an expanding FRW universe. So far an analytic solution has not been found, but our coupled equations of motion are in a form amenable to numerical simulation.

We expect self-intersections to be the dominant route by which cosmic string loops disappear. In [10] we numerically simulated cosmic string loops with constant current around the loop, examining them for self-intersections. It was found that loops with near-maximal current generally did not self-intersect. However, it is to be expected that the current will vary over causally disconnected regions of a loop, so it is natural to ask how a varying current will affect these results. We have adapted this method to examine numerically a general class of loops with varying current in section 3.

In section 4 we discuss what value the current on the string will take. Since the probability of the loop self-intersecting, and hence the lifetime of the loop, is strongly dependent on the size of the current, this is of particular importance. We find that the crucial factor is whether the current switched on when the string was formed, or whether it condensed at a subsequent phase transition. We then look at several possible candidates for the fermions which are trapped on the string. The possibilities considered are a neutrino, or a supersymmetric combination of a gaugino and Higgsino.

Our conclusions are discussed in section 5.

2 Background

We start with an effective action for chiral cosmic strings which is invariant under space-time and worldsheet coordinate transformations, and which constrains the string current to be null. Such an action is given by Carter and Peter [6].

$$S = -\int d^2\sigma \sqrt{-\gamma} \left[m^2 - \frac{1}{2} \psi^2 \gamma_{ij} \phi^{,i} \phi^{,j} \right]$$
 (1)

where γ_{ij} is the metric of the string worldsheet. This is a generalisation of the Nambu-Goto action, containing an extra term due to the current.

The action is still invariant under the transformations

$$\phi \to \tilde{\phi}(\phi)$$
 $\psi \to \tilde{\psi} = \left(\frac{d\phi}{d\tilde{\phi}}\right)\psi$ (2)

and we remove this by a choice of gauge. The physical current on the string must be null for the string to be chiral. It is also conserved and invariant under such transformations, so that it is not the Noether current but rather $j^i = \psi \phi^{,i}$.

2.1 Equations of motion in a general background

The method used to derive the equations of motion in a general background follows the same lines as [10], and also [12]. Varying the action (1) with respect to ψ and ϕ we get, respectively,

$$\sqrt{-\gamma} \,\psi \,\gamma_{ij} \,\phi^{,i} \phi^{,j} = 0 \qquad \qquad D_i(\psi^2 \phi^{,i}) = 0 \tag{3}$$

where the first equation ensures the current is null.

In 1+1 dimensions a scalar field whose gradient is everywhere null must be harmonic, so that the second condition becomes

$$\phi^{i}\psi_{,i} = 0 \tag{4}$$

We fix our gauge by choosing the coordinates on the string worldsheet; the same choices as in [10] are still valid. The timelike coordinate is $\eta = m^{-1}\phi$, and the second coordinate q is taken to be null. The metric then takes the form

$$\gamma_{ij} = \begin{pmatrix} A & B \\ B & 0 \end{pmatrix} \qquad \gamma^{ij} = \begin{pmatrix} 0 & B^{-1} \\ B^{-1} & -AB^{-2} \end{pmatrix} \tag{5}$$

and (3) ensures that $\frac{\partial \phi}{\partial q} = 0$, so that $\psi = \psi(\eta)$, using (4).

Varying the action with respect to x^{ρ} , and dividing by m^2 , then gives :

$$0 = \left[-AB^{-2} + \psi^2 B^{-2} \right] \left(\Gamma^{\rho}_{\mu\nu} x^{\mu}_{,q} x^{\nu}_{,q} + x^{\rho}_{,qq} \right) + 2B^{-1} \left(\Gamma^{\rho}_{\mu\nu} x^{\mu}_{,q} x^{\nu}_{,\eta} + x^{\rho}_{,q\eta} \right)$$
$$-B^{-1} x^{\rho}_{a} \partial_{a} (AB^{-1}) + \psi^{2} B^{-1} x^{\rho}_{a} \partial_{a} (B^{-1})$$

We fix the residual gauge freedom by choosing $A = \psi^2$. The equations of motion then simplify to give a geodesic equation

$$0 = x_{,q\eta}^{\rho} + \Gamma_{\mu\nu}^{\rho} x_{,q}^{\mu} x_{,\eta}^{\nu} \tag{6}$$

Since Nambu-Goto strings are just a special case of chiral cosmic strings, it is unsurprising that this is what is obtained. For, in the Nambu-Goto case we would get a general geodesic equation without any gauge constraints; the absence of a chiral current is reflected in the fact that the coordinates are pure light-cone, since A=0 in this case.

We look at the equations (6) in an expanding universe with conformal time.

$$0 = x_{,q\eta}^{0} + \frac{\dot{a}}{a} \left[x_{,\eta}^{0} x_{,q}^{0} + \mathbf{x}_{,q} \cdot \mathbf{x}_{,\eta} \right]$$
 (7)

$$0 = \mathbf{x}_{,q\eta} + \frac{\dot{a}}{a} \left[\mathbf{x}_{,\eta} x_{,q}^0 + \mathbf{x}_{,q} x_{,\eta}^0 \right]$$
 (8)

The gauge conditions are now

$$A = \psi(\eta)^2 = a(\tau)^2 [\tau_{,\eta} \tau_{,\eta} - \mathbf{x}_{,\eta} \cdot \mathbf{x}_{,\eta}]$$

$$0 = a(\tau)^2 [\tau_{,q} \tau_{,q} - \mathbf{x}_{,q} \cdot \mathbf{x}_{,q}]$$

$$(9)$$

$$(10)$$

In spherical polar coordinates the equations of motion become:

$$0 = \tau_{,q\eta} + \frac{\dot{a}}{a} \left[\tau_{,q} \tau_{,\eta} + r_{,q} r_{,\eta} + r^2 \phi_{,q} \phi_{,\eta} \right]$$

$$0 = r_{,q\eta} + \frac{\dot{a}}{a} \left[r_{,q} \tau_{,\eta} + r_{,\eta} \tau_{,q} \right] - r \phi_{,q} \phi_{,\eta}$$

$$0 = \phi_{,q\eta} + \frac{\dot{a}}{a} \left[\tau_{,q} \phi_{,\eta} + \tau_{,\eta} \phi_{,q} \right] + \frac{1}{r} \left[r_{,q} \phi_{,\eta} + r_{,\eta} \phi_{,q} \right]$$

where ϕ is the azimuthal angle, and we are looking for solutions with $\theta = \frac{\pi}{2}$. So far we have yet to find an analytic solution to these equations. These coupled equations could be studied numerically, though this is outside the scope of this paper.

The rest of this paper deals with chiral strings in flat spacetime.

2.1.1 The equations in Minkowski Space

In flat spacetime the equations of motion reduce to the wave equation [10]. We use a temporal gauge $t = \frac{1}{2}(q+\eta)$, and define $\sigma = \frac{1}{2}(q-\eta)$ where σ measures equal energy intervals along the string.

The general solution is then of the form

$$\mathbf{x}(\eta, q) = \frac{1}{2} [\mathbf{a}(t+\sigma) + \mathbf{b}(t-\sigma)]. \tag{11}$$

Examining the original gauge constraints, which relate to the metric elements, we find that they have become

$$\mathbf{\acute{a}}^2 = 1 \qquad \qquad k^2 \equiv \mathbf{\acute{b}}^2 \le 1 \tag{12}$$

where 'denotes differentiation with respect to the argument.

The current is $j^q = m\psi B^{-1}$ $j^{\eta} = 0$ which gives the physical current as

$$j^t = j^\sigma = \frac{m\psi}{2B} \tag{13}$$

and the conserved charge on the string is

$$N = \int d\sigma \sqrt{-\gamma} j^t = \int d\sigma m \psi(\sigma) = \frac{m}{2} \int d\sigma \sqrt{1 - k^2}$$
 (14)

Obviously there is only one independent conserved quatum number in the chiral case since there is only a left or right moving fermion zero mode in the string core. This is unlike the general case of a current-carrying string where there are two conserved quantities, N and Z, being combinations of the left and right moving currents.

The Nambu-Goto case, which has k=1, thus has zero charge, as required for consistency. However, unlike the Nambu-Goto case, for a string with non-zero current everywhere, there are no cusps $(|\dot{\mathbf{x}}| \neq 1)$. Furthermore, from the metric we see that the physical length l can be found from

$$dl = \sqrt{\gamma_{\sigma\sigma}} d\sigma = \frac{1}{2} \left[1 + k^2 - 2\mathbf{\acute{a}} \cdot \mathbf{\acute{b}} \right]^{1/2} d\sigma. \tag{15}$$

So for k = 1 Nambu-Goto strings $\mathbf{\acute{a}} = -\mathbf{\acute{b}}$ and $dl = d\sigma$, while for k = 0 $dl = d\sigma/2$. As we have shown previously [10], the k = 0 leads immediately to the production of stable loops or vortons.

3 Intersection of varying current chiral strings in a flat background

Since chiral strings have a current that tends to straighten them out, is it still possible for them to self-intersect? In [10] we discussed this numerically for the case of loops with constant current in a flat background. We found that the intersection probability was only substantially affected by near-maximal current, and that the current needed to be larger for more wiggly loops to have the same effect.

The case of loops with varying current was first examined in [13], using a specific type of loop and current. Recall that the charge on the string is proportional to $\sqrt{1-k^2}$. It was found that as the number of points at which k was zero, which correspond to maximal local current, increased the probability of intersection decreased. However, owing to the nature of the loop used, it was impossible to determine whether

it was the number of points with maximal local current, or simply the number of regions of high current, which determined the string self-intersection probability.

We have now carried out a general analysis for k of the type $k(\eta) = \alpha + \beta \cos(n\eta)$, with α and β constants chosen so that $k(\eta)^2 \leq 1$. In fact extra cosine and sine terms can be added to k, with appropriate changes to the code, but this was not examined here as the important features are clear from the simpler case. We generated string loops from series of odd harmonic terms in q and η , using a modified version of the code from [14]. The highest harmonic terms determine how 'wiggly' the string is. Note that this method generates string loops in their centre of mass frame, so that there are no constant terms in $\dot{\mathbf{x}}$.

The parameters chosen for the computation were the same as in [10] and [14], as the results were stable to reasonable modifications. Furthermore, when the amplitude of variation in k was small, the results were comparable to those for constant k, as would be expected.

The results are shown below. We see that the intersection probability is basically unchanged from the constant case if the amplitude of k is small compared to its average. If, however, the amplitude is comparable to, or even greater than, the average value of k, then in general the probability of self-intersection decreases as the frequency increases.

An explanation for this is that regions of high current have more constrained motion. From (11) we have that

$$\dot{\mathbf{x}}(t,\sigma) = \frac{1}{2}[\dot{\mathbf{a}} + \dot{\mathbf{b}}], \qquad \mathbf{x}' = \frac{1}{2}[\dot{\mathbf{a}} - \dot{\mathbf{b}}]. \tag{16}$$

and so $\dot{\mathbf{x}}$ and \mathbf{x}' are roughly parallel when $k^2 \equiv \dot{\mathbf{b}}^2$ is small, which corresponds to high current. If the string has points of maximal current, then the motion is exactly tangential to the string there. For such a string to self-intersect, either the self-intersection has to occur between two adjacent maximal-current points, or else the string has to have such a shape that it can intersect over larger regions. This second type of intersection is suppressed compared to the fixed k case: for, in the fixed current case there are no points of fixed motion preventing certain types of intersection. These large-loop intersections are also constrained, although to a slightly lesser extent, if the current is near-maximal rather than maximal. The size of such a 'near-maximal' current needs to be is determined by the wiggliness of the original string, and it must be larger for more wiggly strings. These features are clearly seen in the graphs.

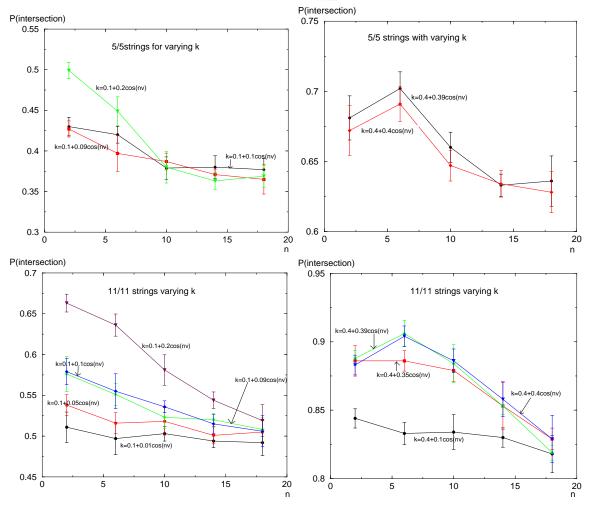
Furthermore it is to be expected that this would be more pronounced for low-harmonic strings, the rationale being that low-harmonic strings have fewer small-scale wiggles and so tend to have more of the suppressed 'large' intersections. This is also seen, if we look at the ratio between strings with the same average current and amplitude which intersect with low frequency k to those with high frequency. Interestingly, though perhaps not surprisingly, the absolute decrease in the probability of intersection is about 0.05 in all cases for a current of $k = 0.1 + 0.1\cos(nv)$, and is similarly fixed in other cases. This may be accounted for by assuming about 5 percent of N/N harmonic loops for any N intersect via these 'large' loop intersections. For

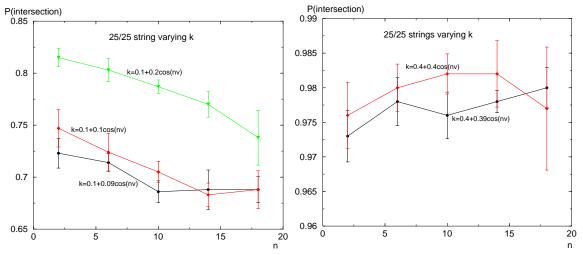
 $k = 0.1 + 0.2\cos(nv)$ the figure is about 15 percent as the minimal current is smaller, and so the loop can curve more, and hence self-intersect on larger scales more easily.

One other thing that stands out from the result is that the intersection probability decreases more slowly, or even increases between n=2 and n=6. This is probably due to the fact that in this model the current adds higher harmonics to the string, although these are suppressed by a factor of the magnitude of the current. Thus the effect is substantially more pronounced for larger currents. At higher frequencies this is negligible in comparison to the stabilising effect of the maximal-current regions.

The only exceptions to this trend are the cases $k = 0.4 + 0.4\cos(nv)$ and $k = 0.4 + 0.39\cos(nv)$ for 25/25 harmonic strings. In this case the strings are extremely wiggly on small scales, and 'larger' intersections do not contribute substantially.

Finally, it appears that the intersection probability is also determined by the maximal value of $k \equiv k_{max}$, so that (at least for small n), the probability of intersection is close to that of the fixed k case with $k = k_{max}$.





From top to bottom, varying k for 5/5, 11/11 and 25/25 strings

4 Properties of chiral cosmic strings

4.1 The current at formation

We have not yet discussed when the current switches on, or what its value is when it condenses. It is possible for the current to condense as the string is formed, for instance in the case of chiral strings resulting from D-term supersymmetric models. However, this need not be the case, and the current may only switch on at a subsequent phase transition.

The value of k at formation, and hence the charge, is in fact closely determined by when the current switches on. For a chiral loop, the number of fermions N on the loop will be given by $N \approx L_{phys}/\lambda$ [11] where λ is the wavelength of the fermion. If the fermion condenses at temperature T_{σ} then we expect $\lambda \sim T_{\sigma}^{-1}$ (in fact this should be a very good approximation).

Using (14) and (15), we have the relationship

$$N = m \int dl \sqrt{1 - k^2} \left[1 + k^2 - 2\mathbf{\acute{a}} \cdot \mathbf{\acute{b}} \right]^{-1/2} \sim L_{phys} T_{\sigma}$$
 (17)

Let us consider the case of constant k for simplicity. The term $\mathbf{\acute{a}} \cdot \mathbf{\acute{b}} = k \cos \theta$ for some θ will average to roughly zero over the loop.

If the current switches on at the string formation temperature $T_X \approx m$, then $T_{\sigma} = T_X \approx m$, and hence we get

$$\sqrt{1-k^2} \int dl \left[1 + k^2 - 2k \cos \theta \right]^{-1/2} \sim L_{phys}$$

The solution to this is obviously $k \approx 0$.

If the current switches on well after the string formation temperature, so that $T_{\sigma}/m \equiv \epsilon \ll 1$, then we get

$$\sqrt{1-k^2} \int dl \left[1 + k^2 - 2k \cos \theta \right]^{-1/2} \sim L_{phys} \epsilon.$$

We try an expansion $k = 1 - \delta$, with $\delta << 1$:

$$L_{phys}\epsilon \sim \sqrt{2\delta - \delta^2} \int dl \left[2 - 2\delta - 2(1 - \delta)\cos\theta + \mathcal{O}(\delta^2) \right]^{-1/2}$$
$$= \delta^{1/2} \left(1 - \frac{\delta}{4} + \mathcal{O}(\delta^2) \right) (1 - \delta + \mathcal{O}(\delta^2))^{-1/2} \int \frac{dl}{\sqrt{1 - \cos\theta}}$$

Note that the integral will contribute a numerical factor of order unity. The denominator diverges like $1/\theta$ for small θ , but this is removed by the $\mathcal{O}(\delta^2)$ corrections. Hence:

$$\epsilon \sim \delta^{1/2} \left[1 + \frac{3}{4} \delta + \mathcal{O}(\delta^2) \right]$$
 (18)

If the current switches on a long time after the string forms, then the left-hand side is small, and so $k \approx 1$. Thus in this case the current at formation is small. Of-course, the current will build up as the loop shrinks, and vorton formation is still possible, though the resulting vortons will be smaller than in the case where $k \approx 0$, and the constraints on the underlying theory less severe [11]

4.2 Daughter loops

If a cosmic string intersects, it will normally be expected to produce daughter loops (eg see [1]) as the strings may intercommute while crossing. This need not be the case, for if there is a topological obstruction to them reconnecting the strings will entangle, with another string forming to join them together. In that case the strings behave very differently, and they will not be considered further here. Note that intercommutation is necessary to obtain the usual string scaling solution.

In general each daughter loop thus formed will possess a kink at the original point of intersection, owing to the discontinuities in the derivatives of \mathbf{a} and \mathbf{b} . These kinks will each split into two kinks moving in opposite directions along the string, the right-hand one moving at the speed of light, while the left-mover travels at subluminal velocity for k < 1. Owing to the rapid acceleration of charge at a kink, it is anticipated that the kinks will quickly evaporate due to radiation, so that in a sense the current smooths out the functions \mathbf{a} and \mathbf{b} .

In the chiral case, the obvious question is: how is the current on the daughter loop related to that of its parent? It seems likely that the current on the daughter loops would be similar to that of their parent. For, the current is determined by the value of k on the loop, which is itself determined by the shape of the loop. Just after the formation of the daughter loops, each daughter will have the same shape as the region of the parent loop from which it came, as this is required by causality. The only difference is that the period of the daughter loop is different from that of its parent. Thus, apart from the kink region, the daughter loops will each evolve according to the equations of motion (6) subject to the gauge conditions (12), with the initial conditions set when the daughter loop forms. The left and right-moving modes, $\bf a$ and $\bf b$, evolve independently and the amplitude of $\bf b'$, k, is unchanged.

The only problem is at the kinks, but these should be smoothed away by radiation. Consequentially the value of k around a kink, which may be discontinuous before, should be averaged out. Of course, as the daughter loop shrinks, the current on the loop will increase since the charge, N, is conserved.

4.3 The fermion on the string

So far we have just specified that there is a single fermion zero mode trapped in the string core of the chiral string. What exactly could this fermion be?

In the N=1 supersymmetric model with a U(1) gauge symmetry the field giving rise to the string is that of the complex scalar part of the primary charged chiral superfield. By considering how the fermionic sector of the theory transforms under supersymmetry it can be shown [15] that the zero mode is a combination of a gaugino and Higgsino. In this case, the current forms at the same time as the string, and by the above argument the current will be large. Typically this is the case in D-term inflation [5], so the resulting cosmic strings are in fact chiral strings. Indeed, the strings form at the end of inflation [4], so this class of models are very likely to have a vorton problem, unless the scale of symmetry breaking is sufficiently small [11]. It might be thought that the zero modes may not survive supersymmetry breaking. However, it was shown in [17] that the chiral zero mode does survive, since there are no other zero modes for it to mix with to become a bound state.

Another possibility is a neutrino zero mode. Neutrinos have very small masses, which are usually explained in terms of the see-saw mechanism. Right-handed neutrino singlet states are introduced, and they are each given a large Majorana mass M_R , while the left-handed neutrinos have zero Majorana mass. This mass would be expected to arise from a Grand Unified Theory undergoing a phase transition, though it can also be put in by hand into the Standard Model. There is also a Dirac mass term M_D , which can arise in the usual way from coupling to the Standard Model Higgs. It can then be shown that the left-handed neutrinos gain a mass of order $m_{\nu} = M_D \frac{1}{M_R} M_D^T$ in matrix form for the three fermion families. The neutrino current could condense on the string at a phase transition after the string formation in this case.

In SO(10) GUTs there are both cosmic strings and right-handed neutrinos, which are the only fermions acquiring a mass from the breaking of the SO(10) symmetry. Above the electroweak scale it may be shown, using the index theorem of [16], that each of the right-handed neutrinos will contribute zero modes to the string, and the string is a chiral string. However, at the electroweak phase transition the ν_R mix with the usual ν_L , giving them small masses as above. The zero modes then cease to be zero modes, becoming instead low-lying bound states. Again this can be seen using the index theorem [16]. In this case the string would no longer be a chiral string. However, it is possible for one of the ν_L to be strictly massless after the electroweak phase transition, and furthermore to be a flavour eigenstate so that it does not mix with the other neutrinos. In this case the corresponding ν_R would remain a zero mode and the string a chiral string. The underlying GUT theory is subject to the constraints in [11].

5 Discussion

In this paper we have extended our study of chiral cosmic strings and their properties. Although string solutions in an expanding Universe have not yet been found, this is to be expected as the same problem occurs in Nambu-Goto strings. Nevertheless the equations of motion have been presented in a way that will hopefully be tractable numerically, if not analytically.

We have presented a heuristic argument for the size of the current. In fact, this provided a strong motivation for saying that the current is near-constant, even across causally disconnected regions of the string, for at each point on the string the current is determined by when the current switched on in relation to the string being formed at that point. Furthermore, the frequency of oscillation of the current will depend on the size of the causally connected regions, where here the timescale is that of the condensation of the current on the string.

It was also found that having a current that can vary around the string loop does not change the fundamental conclusion that for near-maximal currents the strings are not expected to self-intersect, while for lower currents, such self-intersections are extremely likely. Thus the idea of large vortons remains, and indeed the over-production of such things, which would come to dominate the energy density of the Universe, provides a constraint on all models that predict such chiral strings. This suggests that models based on D-term inflation, with chiral strings formed at the end of inflation, are strongly constrained by the overproduction of vortons and are unlikely to be viable models for structure formation.

Finally we looked at how these chiral strings might arise. The most convincing mechanism is that of a string formed by D-term SUSY breaking, as in this case the current is expected to be large, resulting in vorton production. Vortons may also have a role to play in ruling out neutrino models, although this is more tentative at the moment.

6 Acknowledgements

This work is supported in part by PPARC and an ESF network. We are grateful to Tom Kibble and Rachel Jeannerot for discussions.

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